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A Partial Safety Factor Method for System Reliability Prediction with Outsourced Components

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Abstract

System reliability is usually predicted with the assumption that all component states are independent. This assumption may not accurate for systems with outsourced components since their states are strongly dependent and component details may be unknown. The purpose of this study is to develop an accurate system reliability method that can produce complete joint probability density function (PDF) of all the component states, thereby leading to accurate system reliability predictions. The proposed method works for systems whose failures are caused by excessive loading. In addition to the component reliability, system designers also ask for partial safety factors for shared loadings from component suppliers. The information is then sufficient for building a system-level joint PDF. Algorithms are designed for a component supplier to generate partial safety factors. The method enables accurate system reliability predictions without requiring proprietary information from component suppliers.

1. Introduction

System reliability is the ability that a system performs its intended function. It is often measured by the probability that the system can work properly without any failure. Since a system is composed of multiple components, its reliability depends on the reliability of each component. Accurate system reliability prediction requires the joint probability density function (PDF) of all the component states. It is difficult or even impossible to obtain the joint PDF. For this reason, the system reliability is commonly approximated with the assumption that all component states are independent.

In this work, we mainly focus on series systems since they are very common in engineering applications. For a series system consisting of *n* components, the assumption gives

$$R_s = \prod_{i=1}^{n} R_i \tag{1}$$

where R_s is the system reliability, and R_i is the reliability of component *i*.

The independence assumption is particularly useful for systems whose components are outsourced. Outsourcing is a common practice as many industrial firms, such as automakers, function as system integrators, relying on various outside component suppliers. For example, numerous parts of vehicles are designed and manufactured outside except engines and powertrains that the automaker wants to keep in-house. This practice has resulted in huge cost savings in developing new products [1, 2]. During the system design stage, system designers can easily estimate the system reliability using Eq. (1) after they obtain component reliability R_i ($i = 1, 2, \ldots, n$) from component suppliers. The independence assumption does not require system designers to know component design details [3, 4], which in most cases are proprietary to component suppliers. When the component reliability is predicted with physics-based reliability methods, the component design details include component limit-state functions that specify the component states (safe or failed) [5].

The major drawback of the independence assumption method is the poor accuracy when component states are strongly dependent. This is often the case for mechanical systems. Components in a mechanical system may share the same random operation conditions, such as excessive stresses, making component failures highly dependent. Eq. (1) is actually the worst-case system reliability when component states are positively dependent. (This is the case for most mechanical applications.) The best-case system reliability is equal to the worst component reliability $\min\{R_i\}_{i=1,2,\ldots,n}$ under the assumption that all component failures are completely dependent. Then the error of the system reliability prediction without knowing the system joint PDF is given by [6]

$$\prod_{i=1}^{n} R_i \leq R_s \leq \min\{R_i\}, \quad (i = 1, 2 \cdots, n) \tag{2}$$

The above reliability bound may be too wide to make any useful decisions. To narrow this bound, Ditlevsen [7] proposed a method to obtain series system reliability bounds with the involvement of both unicomponent probabilities and bicomponent probabilities. Zhang [8] generalized this method by introducing joint probabilities of larger sets of components. Both methods require complete limit-state functions of all the components, making them not applicable for systems with outsourced components. Song and Der Kiureghian [9, 10] applied Linear Programming (LP) to compute the optimal bounds for system reliability based on any level of given information, such as incomplete component probabilities or inequality constrains on component probabilities. An approximate LP algorithm, however, may not perform well for over-constrained problems. Then based on the LP algorithm, Kang and Song developed a matrix-based system reliability (MSR) method [11], which makes it possible to produce narrow bounds for the probability of system failure. Simplifying the system event in a matrix form, the MSR method can estimate system reliability accurately even in the case where the component probabilities and/or the statistical dependence are incomplete. This method still requires the distributions of input random variables and may not work well for systems with outsourced components.

To address this issue, Hu and Du [12, 13] proposed a physics-based reliability method for component adopted in new series systems. The method reconstructs component limit-state functions at the system-level using limited reliability information. This method is able to build the joint PDF of the component states, thereby estimating system reliability with high accuracy. It requires, however, reliability functions with respect to the system load, which increases the burden of component reliability analysis on the component supplier side. To fix this problem, a new method [14] was developed to rebuild an equivalent component limit-state function under new conditions without knowing the relationship between the reliability and load. But this method may be inefficient for systems with more than two shared loads among components.

Many other reliability methods can also be used for the system reliability prediction. Yu and Wang [15] proposed a reliability assessment approach by combining the extreme value moment method and the improved maximum entropy method for systems with multiple failure modes. Recently, they also developed a novel time-variant reliability analysis method based on failure process decomposition for dynamic systems [16] and a kernel density function based on the uncertainty quantification method for estimating the reliability of a robotic device [17]. Youn and Wang [18] developed a complementary intersection method (CIM) for series system reliability analysis. CIM makes it possible to use common reliability methods for efficiently calculating the probability of high-order joint failure events. This method was then extended to a more general form, referred to as the generalized complementary intersection method (GCIM) [19], and it can be used for parallel and mixed systems. For system reliability assessment with multiple dependent failure events, an integrated performance measure approach (iPMA) was proposed [20]. iPMA employs Gaussian process regression to construct component surrogate models, which then enables system reliability prediction directly using Monte Carlo Simulation (MCS). However, if the component limit-state functions or exact distributions of design variables are not available, these methods will not be applicable for system reliability evaluation with outsourced components.

Some statistical-based methods are also widely used for system reliability evaluation, including Kriging Surrogate Modeling [21, 22], Linear Regression (LR) [23], Artificial Neural Network (ANN) [24], and Support Vector Machines (SVM) [25, 26]. Even with no component design details, these methods could reconstruct a precise decision boundary (response surface) of the component using training data. To evaluate the system reliability, however, they still require additional information from component suppliers.

The objective of this work is to develop a new system reliability method linking both component-level and system-level analyses. At the component level, the proposed method enables component suppliers to provide enough information to system designers without revealing their component design details. At the system level, the proposed method helps system designers produce a complete joint PDF of all the component states, thereby leading to accurate system reliability prediction. Specifically, the major approach we use in the proposed method is the employment of partial safety factors (PSFs), which are specified by component suppliers for shared loads from the system with physics-based reliability. Then system designers use the PSFs from component suppliers to rebuild equivalent component limit-state functions [27-29], which in turn produce the joint PDF that is necessary for the system reliability prediction.

The rest of this paper is organized as follows. Basic methodologies used in this work are reviewed in Section 2. The overview of the proposed methods is given in Section 3. The system-level analysis is discussed in Section 4 followed by component-level analysis in Section 5. In Section 6 the complete procedure of the proposed method is described. Two examples are discussed in Section 7. Conclusions are given in Section 8.

2. Methodology Review

The proposed method can employ any physics-based reliability methods, including First-Order Reliability Method (FORM), Second-Order Reliability Method (SORM), and MCS. The methods are briefly reviewed in Section 2.1. We also review the concept of PSF in Section 2.2.

2.1 First Order Reliability Method (FORM)

FORM linearizes a limit-state function $g(\mathbf{X})$ using the first order Taylor expansion.

Step 1: Transform random variables into standard normal variables

Assume that all random variables in $\mathbf{X}=(X_1, X_2, \cdots, X_n)$ are independent. The random variables in $\mathbf{X}$ are transformed into standard normal random variables $\mathbf{U}=(U_1, U_2, \cdots, U_n)$ by [30]

$$F_i(X_i)=\Phi(U_i) \quad (i=1,2,\cdots,n) \tag{3}$$

where $F_i(\cdot)$ and $\Phi(\cdot)$ are the cumulative distribution functions (CDF) of X_i and U_i, respectively. Then

$$X_i = F^{-1}(\Phi(U_i)) = T(U_i) \quad (i=1,2,\cdots,n) \tag{4}$$

in which $T(\cdot)$ denotes the transformation operation.

Step 2: Search for the Most Probable Point (MPP)

p_f is computed by

$$p_f = \Pr\{g(T(\mathbf{U})) < 0\} = \int\limits_{g(T(\mathbf{U}))<0} \phi_{\mathrm{U}}(\mathbf{u})d\mathbf{u} \tag{5}$$

in which $\phi_{\mathrm{U}}(\mathbf{u})$ is the joint PDF of $\mathbf{U}$.

FORM linearizes $g(T(\mathbf{U}))$ using an expansion point $\mathbf{u}^*$ obtained from

$$\begin{cases} \min\limits_{\mathbf{U}} \ \sqrt{\mathbf{U}\mathbf{U}^T} \\ \text{s.t. } \ g(T(\mathbf{U})) = 0 \end{cases} \tag{6}$$

$\mathbf{u}^*$ is the MPP, and its magnitude is called the reliability index and is given by

$$\beta = \sqrt{\mathbf{u}^*\left(\mathbf{u}^*\right)^T} \tag{7}$$

With the first Taylor expansion series, $g(T(\mathbf{U}))$ is approximated at $\mathbf{u}^*$ as

$$g(T(\mathbf{U})) \approx g(T(\mathbf{u}^*)) + \nabla g(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*)^T = \nabla g(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*)^T \tag{8}$$

where $\nabla g(\mathbf{u}^*)$ is the gradient of $g(T(\mathbf{U}))$ at $\mathbf{u}^*$ and is given by

$$\nabla g(\mathbf{u}^*) = \left(\frac{\partial g(T(\mathbf{U}))}{\partial U_1}, \frac{\partial g(T(\mathbf{U}))}{\partial U_2}, \cdots, \frac{\partial g(T(\mathbf{U}))}{\partial U_n} \right)\Bigg|_{\mathbf{u}^*} \tag{9}$$

Set a unit vector $\boldsymbol{\alpha}$ as

$$\boldsymbol{\alpha} = \frac{\nabla g(\mathbf{u}^*)}{\| \nabla g(\mathbf{u}^*) \|} \tag{10}$$

Then $\mathbf{u}^*$ is represented by

$$\mathbf{u}^* = -\beta\boldsymbol{\alpha} \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (8) and multiplying both sides of Eq. (8) by $\frac{1}{\| \nabla g(\mathbf{u}^*) \|}$ yields a new limit-state function

$$G(\mathbf{U}) = \frac{g(T(\mathbf{U}))}{\| \nabla g(\mathbf{u}^*) \|} \approx \beta + \boldsymbol{\alpha}\mathbf{U} \tag{12}$$

Step 3: Compute p_f

Using the new limit-state function in Eq. (12), p_f is calculated by

$$p_f \approx \Pr\{G(\mathbf{U}) < 0\} = \Phi(-\beta) \tag{13}$$

Since FORM is based on the first order Taylor expansion, it is accurate when the component limit-state function is not highly nonlinear. Otherwise, SORM may be a better choice.

2.2 Second Order Reliability Method (SORM)

SORM uses the second order Taylor expansion to approximate $g(\mathbf{X})$ at the MPP, which is given by

$$\begin{aligned} g(T(\mathbf{U})) \approx g(\mathbf{u}^*) + \nabla g(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*) \\ + \frac{1}{2}(\mathbf{U} - \mathbf{u}^*)\nabla^2 g(\mathbf{u}^*)(\mathbf{U} - \mathbf{u}^*) \end{aligned} \tag{14}$$

where $\nabla g(\mathbf{u}^*)$ is given in Eq. (9) and $\nabla^2 g(\mathbf{u}^*)$ is the Hessian matrix. Since there is no closed-form expression for p_f [31], an orthogonal transformation $\mathbf{Y} = \mathbf{HU}$ is conducted. This transformation rotates the U-space into a new set of mutually independent standard normal variables $\mathbf{Y}$ with Y_n coincident with the MPP vector. Matrix $\mathbf{H}$ is an orthogonal matrix and is obtained by a Gram-Schmidt [32] orthogonalization. Then the approximated limit-state function is rewritten as

$$G(\mathbf{Y}) \approx -Y_n + \beta + \frac{1}{2}(\mathbf{Y} - \mathbf{y}^*)\mathbf{M}(\mathbf{Y} - \mathbf{y}^*)^T \tag{15}$$

where $\mathbf{y}^* = (0, 0, \cdots, \beta)^T$ is the Y-space MPP corresponding to the $\mathbf{u}^*$, and $\mathbf{M}$ is the transformed Hessian matrix and is given by

$$\mathbf{M} = \mathbf{H}\frac{\nabla^2 g(\mathbf{u}^*)}{\|\nabla g(\mathbf{u}^*)\|}\mathbf{H}^T \tag{16}$$

After a series of orthogonal transformations, with the first $n-1$ variables being $\overline{Y}=(y_1, y_2, \cdots, y_{n-1})^T$, the first $(n-1)\times(n-1)$ order matrix of $\mathbf{M}$ becomes a diagonal matrix, and Eq. (15) becomes

$$Y_n = \beta + \frac{1}{2}\sum_{i=1}^{n-1} k_i y_i'^2 \tag{17}$$

where k_i represents the curvature of the response surface at the MPP, and finding k_i can be treated as an eigenvalue problem.

The probability of failure is then estimated using Breitung's formulation, which is given by

$$p_{f,\,\text{Breitung}} = \Phi(-\beta)\prod_{i=1}^{n-1}(1+k_i\beta)^{-1/2} \tag{18}$$

A more accurate expression is derived from Tvedt's formulations which is given by [32]

$$p_{f,\,\text{Breitung}} = A_1 + A_2 + A_3 \tag{19}$$

in which

$$\begin{cases} A_1 = \Phi(-\beta)\prod_{i=1}^{n-1}(1+k_i\beta)^{-1/2} \\ A_2 = [\beta\Phi(-\beta) - \Phi(\beta)] \\ \qquad \times\left[\prod_{i=1}^{n-1}(1+k_i\beta)^{-1/2} - \prod_{i=1}^{n-1}(1+k_i(\beta+1))^{-1/2}\right] \\ A_3 = (\beta+1)[\beta\Phi(-\beta) - \Phi(\beta)] \\ \qquad \times\left[\prod_{i=1}^{n-1}(1+k_i\beta)^{-1/2} - \mathrm{Re}\left[\prod_{i=1}^{n-1}(1+k_i(\beta+1))^{-1/2}\right]\right] \end{cases} \tag{20}$$

where $\mathrm{Re}(\cdot)$ denotes the real part of an imaginary number.

2.3 Monte Carlo Simulation (MCS)

MCS is a sampling method. The procedure of MCS is below.

1) Generate N samples of $\mathbf{X}$.

2) Calculate the response $g(\mathbf{X})$ at samples of $\mathbf{X}$, and then N samples of $g(\mathbf{X})$ are available.

3) Count the number of samples of $g(\mathbf{X})$ in the failure region ($g(\mathbf{X})<0$). Denote the number of failures by N_f. The probability of failure is then given by

$$p_f = \frac{N_f}{N} \tag{21}$$

2.4 Partial Safety Factor (PSF)

PSFs are commonly used in modern structural design (limit state design), which affects the action values (loads) and the characteristic values of the material properties, and the satisfaction of certain ultimate and serviceability limit states [33]. To obtain a safe design, the acting loads and material properties are combined with specified PSFs. The PSF for a load, which is generally greater than unity, sets the design value of the load equal to the product of the PSF and the acting load (or desired load). The PSF for a material strength is usually less than unity. Multiplying the PSF by the material strength determines the permissible stress (strength) of the material. For example, a load acting on a cantilever beam is multiplied by a PSF > 1 to account for the variation of the load due to a sudden increase. Similarly, a PSF < 1 is applied to the characteristic stress of the material to ensure that sufficient strength is provided.

In general, for a component with limit-state function $g(\mathbf{X})$, the basic random variables in $\mathbf{X}$ include applied loads $\mathbf{L}=\left(L_1, L_2, \cdots, L_p\right)$ and component strength S. With PSFs, the safe state of the component is specified by [34]

$$g(\mathbf{X}) = g(\lambda\mu_S, \gamma_i\mu_{L_i}) > 0 \tag{22}$$

where $\lambda < 1$ is the PSF (reduction factor of strength) for the strength, and γ_i is the PSF (partial load amplification factor) for load L_i ($i = 1, 2, \ldots, p$); μ_S is the mean of S, and μ_{Li} is the mean of L_i.

With distributions of S and L_i available, the PSFs λ and γ_i could be easily computed. Assume that component suppliers use FORM for the reliability analysis, which produces the MPP $\mathbf{u}^*$ and the reliability index β. The partial safety factors λ and γ_i can be obtained by

$$\lambda = \frac{S^*}{\mu_S} = \frac{F_S^{-1}\left(\Phi(-\beta\alpha_S)\right)}{\mu_S} \tag{23}$$

$$\gamma_i = \frac{L_i^*}{\mu_{L_i}} = \frac{F_L^{-1}\left(\Phi(-\beta\alpha_{L_i})\right)}{\mu_{L_i}} \tag{24}$$

in which S^* and L_i^* are components of the MPP in in the X-space for S and L_i, respectively; α_S and α_{L_i} are the directional cosine of S and L_i in the U-space, respectively.

If $g(\mathbf{X})$ is not available to component suppliers, physics-based methods cannot be directly applied for the reliability analysis. In these cases, statistics-based methods, such as Support Vector Machine [35], are good choices to approximate the component limit-state function with limited observations; then λ and γ_i will be available.

3. System Reliability Prediction with PSFs

The proposed system reliability method works for the following systems with outsourced components.

1) System and component failures are caused by excessive stresses.

2) Components share a number of loads, which are the only common basic variables shared by component limit-state functions.

3) A component may have multiple failure modes.

4) System designer knows the distributions of loads distributed to components.

5) System designers do not know component limit-state functions.

6) System designers know component reliability provided by component suppliers.

7) Component suppliers also provide PSFs they used in their component design to system designers.

The basic strategy of the PSF method is that system designers construct equivalent component limit-state functions and convert them into a multivariate normal distribution, whose distribution parameters are estimated through the component reliability and component PSFs provided by component suppliers. Once the joint normal PDF is available, the system reliability can be easily estimated.

It is therefore important for component suppliers to produce component reliabilities and PSFs. For the former, any physics-and statistics-based methods, such as FORM, SORM, MCS, SVM, and experiments can be used. For the latter, the proposed method relies on the concept of equivalent linear safety margin [27] to determine PSFs for components with multiple failure modes.

The PSF method therefore involves both system- and component-level reliability analyses. Both of them are discussed in Sections 4 and 5.

4. System-Level Analysis

At the system analysis level, the task of system designers is to accurately predict system reliability with only the component reliability and corresponding PSFs.

Assume that the system consists of m components and is subjected to multiple loads $\mathbf{L}=\left(L_1, L_2, \cdots, L_p\right)$. Component probabilities of failure p_{fi} and PSFs $\gamma_i=\left(\gamma_{i,1}, \gamma_{i,2}, \cdots, \gamma_{i,p}\right)$ of $\mathbf{L}=\left(L_1, L_2, \cdots, L_p\right)$, $i=1,2,\ldots,m$, are provided by component suppliers.

For component *i*, system designers construct an equivalent limit-state function no matter how many failure modes the component may have and what reliability method that the component supplier has used. With FORM, the equivalent limit-state function contains only shared load $\mathbf{L}=\left(L_1, L_2, \cdots, L_p\right)$ in a linear form

$$G_i(\mathbf{U}) \approx \beta_i + \alpha_{i,L_1} U_{L_1} + \cdots + \alpha_{i,L_p} U_{L_p} \tag{25}$$

where β_i is the reliability index given by

$$\beta_i = \Phi(-p_{fi}) \tag{26}$$

$\mathbf{U}_L=(U_{L_1}, U_{L_2}, \ldots, U_{L_p})$ is the transformed vector of $\mathbf{L}=\left(L_1, L_2, \cdots, L_p\right)$, and α_{i,L_j}, $j=1,2,\ldots,p$, are coefficients.

System designers can find α_{i,L_j} using PSFs $\gamma_{i,j}$. The equation is given by

$$\alpha_{i,L_j} = -\beta_i^{-1}\Phi^{-1}\left(F_{L_j}(\gamma_{i,j}\mu_{L_j})\right) \tag{27}$$

Eq. (27) can be easily derived if FORM is used by the component supplier. Since $L_{i,j}^* = \gamma_{i,j}\mu_{L_j}$ is the MPP component of L_i in the X-space, we have

$$F_{L_j}(L_{i,j}^*) = \Phi(u_{L_j}^*) \tag{28}$$

in which $F_{L_j}(\cdot)$ is the CDF of L_j, and $u_{L_j}^*$ is the MPP component of L_j in the U-space. According to Eq. (11) and $u_{L_{i,j}}^* = -\alpha_{i,L_j}\beta_i$, Eq. (27) is rewritten as

$$F_{L_j}(L_j^*) = \Phi(-\alpha_{i,L_j}\beta_i) \tag{29}$$

This leads to Eq. (27).

Note that using FORM is not a prerequisite for the PSF method. As will be discussed in Section 5, other reliability methods can also be used.

Since the components of $\mathbf{U}$ follow standard normal distributions, the limit-state function $G_i(\mathbf{U})$ with respect to $\mathbf{U}$ also follows a normal distribution with the mean value of β_i and standard deviation of 1. Thus, the joint PDF of all the component states in Eq. (25) follows a multivariate normal distribution with the joint PDF $\phi_{\mathbf{G}}(\cdot)$ determined by the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\boldsymbol{\mu}$ is given by

$$\boldsymbol{\mu} = (-\beta_1, -\beta_2, \cdots, -\beta_m) \tag{30}$$

and $\boldsymbol{\Sigma}$ is given by

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1m} \\ \rho_{21} & 1 & & \rho_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{m1} & \rho_{m2} & \cdots & 1 \end{bmatrix}_{m\times m} \tag{31}$$

in which ρ_{ij} is the correlation coefficient between the i-th and j-th component states and is calculated by

$$\rho_{ij} = \sum_{k=1}^{p} \alpha_{i,Lp}\alpha_{j,Lp} \tag{32}$$

With the obtained $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, $\phi_{\mathbf{G}}(\cdot)$ is given by

$$\phi_{\mathbf{G}}(\mathbf{v}) = \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} \exp\left(-\frac{1}{2}(\mathbf{v}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{v}-\boldsymbol{\mu}) \right) \tag{33}$$

The system reliability is calculated by

$$R_s = \int_{\Omega} \phi_{\mathbf{G}}(\mathbf{v})\,\mathrm{d}\,\mathbf{v} = \Phi_m(-\boldsymbol{\mu};\boldsymbol{\Sigma}) \tag{34}$$

where $\Phi_m(-\boldsymbol{\mu};\boldsymbol{\Sigma})$ is the CDF of $\phi_{\mathbf{G}}(\mathbf{v})$, $\mathbf{v}$ is a m-dimensional random vector, and Ω is the system safe region defined by

$$\Omega=\left\{\mathbf{U} \mid -G_i(\mathbf{U})<0 \ (i=1,2,\cdots,m)\right\} \tag{35}$$

The system probability of failure is then given by

$$p_{fs}=1-R_s \tag{36}$$

5. Component-Level Analysis

As discussed above, the task of a component supplier is to provide the component reliability and PSFs of the shared loads to system designers. We now discuss the proposed method for doing so.

A component may fail due to multiple failure modes. For each failure mode the component supplier could use various methods to obtain the component reliability.

Given component i with q failure modes and the limit-state functions $G_{i,k}(\mathbf{U}_i)=g_{i,k}(\mathbf{U}_{i\sim L},\mathbf{U}_L)$ $(k=1,2\cdots,q)$, where $\mathbf{U}_i$ is the vector of the basic variables, $\mathbf{U}_L$ is the vector of the shared loads, and $\mathbf{U}_{i\sim L}$ is the vector of $\mathbf{U}_i$ without $\mathbf{U}_L$ in the U-space. We at first discuss the case where FORM is used. The approximated limit-state functions by FORM are given by

$$G_{i,k}(\mathbf{U})\approx \beta'_{i,k}+\alpha'_{i,k1}U_{i\sim L1}+\cdots+\alpha'_{i,kL_1}U_{iL_1}+\cdots+\alpha'_{i,kL_p}U_{iL_p},\ k=1,2\cdots,q \tag{37}$$

where $\beta'_{i,k}$ is the reliability index of the k-th failure mode, and $\boldsymbol{\alpha}'_{i,k}=(\alpha'_{i,k1},\cdots,\alpha'_{i,kL_1},\cdots,\alpha'_{i,kL_p})$ is the directional cosine. If one failure mode occurs, the entire component fails. As a result, the component is regarded as a series system. The reliability is then given by

$$R_i=\Phi_k(-\boldsymbol{\mu}_i,\boldsymbol{\Sigma}_i) \tag{38}$$

where $\boldsymbol{\mu}_i = (-\beta'_{i1}, -\beta'_{i2}, \cdots, -\beta'_{iq})$ and $\boldsymbol{\Sigma}_i = [\rho_{kj}] = \boldsymbol{\alpha}'_{i,k} \boldsymbol{\alpha}'^{T}_{i,j}$ $(k, j = 1, 2, \cdots, q)$. The reliability index of the component is therefore given by

$$\beta_i = \Phi^{-1}\left(R_i\right) \tag{39}$$

Each component failure mode has its own PSFs for the shared loads. To enable the component supplier to produce PSFs for the entire component with a single limit-state function, we employ the method of the equivalent linear safety margin discussed in [27], which is given by

$$G_i(\mathbf{U}) = \beta_i + \alpha_{i,1} U_{1\sim L} + \cdots + \alpha_{i,L_1} U_{L_1} + \cdots + \alpha_{i,L_p} U_{L_p} \tag{40}$$

Eq. (40) represents only one limit-state function no matter how many failure modes a component may have. The coefficients of the random variables on the right-hand side are determined by the sensitivity of β_i with respect to the basic random variables.

$$\alpha_{i,j} = \frac{\dfrac{\partial \beta_i}{\partial U_j}}{\left(\sum_{j=1}^{n} \left(\dfrac{\partial \beta_i}{\partial U_j}\right)^2\right)^{1/2}},\ j = 1, 2, \ldots, n \tag{41}$$

The derivatives in Eq. (41) are evaluated numerically. Increase U_j, $j = 1, 2, \ldots, n$, by a small step size $\varepsilon_j > 0$, and let $\boldsymbol{\varepsilon}_j = (0, \ldots, \varepsilon_j, \ldots, 0)$. Since U_j is a standard normal variable, ε_j is set to $\varepsilon_j = 0.01$, or one percent of the standard deviation [36], which is used for both examples in this work. The new basic variables then become

$$\mathbf{U} = (U_1, U_2, \ldots, U_{j-1}, U_j + \varepsilon_j, U_{j+1} \ldots, U_n) \tag{42}$$

This gives a new reliability index $\beta_{i,j}$ by

$$\beta_{i,j} = -\Phi^{-1}\left(1 - \Phi(-\beta_i - \boldsymbol{\alpha}_i \boldsymbol{\varepsilon}_j{}^T)\right) \tag{43}$$

The derivative is then given by

$$\frac{\partial \beta_i}{\partial U_j} \approx \frac{\beta_{i,j} - \beta_i}{\varepsilon} \tag{44}$$

Since the counterpart in the X-space of the MPP of L_j is calculated by $L_j^* = \gamma_{i,j}\mu_{L_j}$ and in the U-space the MPP is given by $u_j^* = -\beta_i \alpha_{i,L_j}$, the component PSFs for the shared loads are computed by

$$\gamma_{i,j} = \frac{F_{L_j}^{-1}\left(\Phi(-\beta_i \alpha_{i,L_j})\right)}{\mu_{L_j}} \quad (j = 1, 2, \cdots, p) \tag{45}$$

Then component suppliers provide the component reliability R_i and PSFs $\gamma_{i,j}$ to system designers. R_i and $\gamma_{i,j}$ do not include proprietary information such as component limit-state functions, which may involve component structures, dimensions, and material properties.

Note that although the above discussions are based on FORM, other reliability methods can also be used. If SORM is used, the procedure will be the same. Component suppliers only need to replace β_i obtained by FORM with that by SORM.

6. Complete Procedure

We now discuss the complete procedure of using the PSF method. The procedure consists component-level and stem-level analyses. First, we summarize the information known at both levels.

1) Component-level: the limit-state functions $g_{i,k}(\mathbf{X})$ for the *k*-th failure mode of the *i*-th component, and the distributions of $\mathbf{X}$ (including the basic random variables and system loads).

2) System-level: the probability of failure of each component p_{fi} , the PSF $\gamma_{i,j}$ for each system load, and the distributions of system loads. The former two pieces of information are produced by the component-level analysis.

The flowchart of the complete procedure is then provided as shown in Fig.1.

Place Figure 1 here

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7. Examples

In this section, the PSF method is applied to two examples. The first mathematical example is used to demonstrate the procedure of using the proposed method for system reliability estimation while the other example shows an engineering application.

7.1 A mathematical example

A system consists of two components, and each component has two failure modes (FMs). The components are provided by two different outside suppliers. We now discuss the proposed method through both component-level and system-level analysis.

7.1.1 Component-level analysis

Component 1 has two limit-state functions for FM1 and FM2, respectively, which are given by

$$g_{1,1}(\mathbf{X}_1) = -452 + 8.6X_{1,1} + 3.6X_{1,2} + X_{1,3} \tag{46}$$

$$g_{1,2}(\mathbf{X}_1) = -1035 + X_{1,1}^3 + 2X_{1,2}^2 - 3X_{1,3} \tag{47}$$

The independent basic random variables are $\mathbf{X}_1 = (X_{1,1}, X_{1,2}, X_{1,3}) = (X_{1,1}, \mathbf{L})$, and $\mathbf{L} = (L_1, L_2)$ contains two shared loads. Their distributions are given in Table 1, in which N means Normal Distribution.

Place Table 1 here

The supplier uses FORM for reliability analysis for FM1 and obtains the approximated limit-state function given by

$$G_{1,1}(\mathbf{U}) = \beta'_{1,1} + \alpha'_{1,11} U_{1,1} + \alpha'_{1,1L_1} U_{1L_1} + \alpha'_{1,1L_2} U_{1L_2} \tag{48}$$

where $\beta'_{1,1} = 3.1614$, $\alpha'_{1,11} = 0.5179$, $\alpha'_{1,1L_1} = 0.4065$, and $\alpha'_{1,1L_2} = 0.7527$. The probability of failure is $p_{f1,1} = \Phi(-\beta'_{1,1}) = 7.8498 \times 10^{-4}$.

For FM2, the suppliers applies SORM due to the higher nonlinearity. Then the reliability index and corresponding directional cosine are obtained by $\beta'_{1,2} = 3.3435$, $\alpha'_{1,21} = 0.7012$, $\alpha'_{1,2L_1} = 0.7006$, and $\alpha'_{1,2L_2} = -0.1320$. The approximated linear limit-state function is given by

$$G_{1,2}(\mathbf{U}) = \beta'_{1,2} + \alpha'_{1,21} U_{1,1} + \alpha'_{1,2L_1} U_{L_1} + \alpha'_{1,2L_2} U_{L_2} \tag{49}$$

The probability of failure is $p_{f1,2} = \Phi(-\beta'_{1,2}) = 4.1359 \times 10^{-4}$.

Since the joint PDF of $G_{1,1}(\mathbf{U})$ and $G_{1,2}(\mathbf{U})$ follows multivariate normal distribution with the mean $\boldsymbol{\mu}_1$ given by

$$\boldsymbol{\mu}_1 = (-\beta'_{1,1}, -\beta'_{1,2}) = (-3.1614, -3.3435) \tag{50}$$

and $\boldsymbol{\Sigma}_1$ given by

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.5485 \\ 0.5485 & 1 \end{bmatrix} \tag{51}$$

Thus, the probability of failure of component 1 is calculated by

$$p_{f1} = 1 - \Phi_2(-\boldsymbol{\mu}_1; \boldsymbol{\Sigma}_1) = 1.1650 \times 10^{-3} \tag{52}$$

The corresponding reliability index is $\beta_1 = -\Phi^{-1}(p_{f1}) = 3.0446$.

The component supplier also needs to provide the PSFs for the system load $\mathbf{L} = (L_1, L_2)$ to system designers. Now we discuss how the component supplier obtains the PSFs using the equivalent linear safety margin approach [28].

The component equivalent reliability index is β_1. Set $\varepsilon = 0.01$, with $\boldsymbol{\varepsilon}_1 = (\varepsilon, 0, 0)$, we have

$$\begin{aligned} -\boldsymbol{\beta}_1^T - \boldsymbol{\alpha}_1 \boldsymbol{\varepsilon}_1^T &= \begin{bmatrix} -\beta'_{1,1} \\ -\beta'_{1,2} \end{bmatrix} - \begin{bmatrix} \alpha'_{1,11} & \alpha'_{1,1L_1} & \alpha'_{1,1L_2} \\ \alpha'_{1,21} & \alpha'_{1,2L_1} & \alpha'_{1,2L_2} \end{bmatrix} \begin{bmatrix} \varepsilon \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -3.1614 \\ -3.3435 \end{bmatrix} - \begin{bmatrix} 0.5179 & 0.4065 & 0.7527 \\ 0.7012 & 0.7006 & -0.1320 \end{bmatrix} \begin{bmatrix} 0.01 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -3.1666 \\ -3.3505 \end{bmatrix} \end{aligned} \tag{53}$$

According to Eq. (43), the new reliability index is

$$\beta_{1,1}(\varepsilon_1) = -\Phi^{-1}\left(1 - \Phi_2(-3.1666, -3.3505; \boldsymbol{\Sigma}_1)\right) = 3.0506 \tag{54}$$

Therefore,

$$\alpha_{1,1} = \left.\frac{\partial \beta_1}{\partial \varepsilon}\right|_{\varepsilon_1 = 0} = \frac{\beta_{1,1}(\varepsilon_1) - \beta_1}{\varepsilon} = \frac{3.0506 - 3.0446}{0.01} = 0.6051 \tag{55}$$

Similarly, with $\varepsilon_2 = (0, \varepsilon, 0)$, we have

$$\beta_{1,L_1}(\varepsilon_2) = -\Phi^{-1}\left(1 - \Phi_2(-3.1655, -3.3505; \boldsymbol{\Sigma}_1)\right) = 3.0499 \tag{56}$$

Therefore,

$$\alpha_{1,2} = \alpha_{1,L_1} = \left.\frac{\partial \beta_1}{\partial \varepsilon}\right|_{\varepsilon_2=0} = \frac{\beta_{1,L_1}(\boldsymbol{\varepsilon}_2) - \beta_1}{\varepsilon} = \frac{3.0499 - 3.0446}{0.01} = 0.5292 \tag{57}$$

Likewise, with $\boldsymbol{\varepsilon}_3 = (0, 0, \varepsilon)$, we have

$$\beta_{1,L_2}(\boldsymbol{\varepsilon}_3) = -\Phi^{-1}\left(1 - \Phi_2(-3.1690, -3.3422; \boldsymbol{\Sigma}_1)\right) = 3.0492 \tag{58}$$

Thus,

$$\alpha_{1,3} = \alpha_{1,L_2} = \left.\frac{\partial \beta_1}{\partial \varepsilon}\right|_{\varepsilon_3=0} = \frac{\beta_{1,L_2}(\boldsymbol{\varepsilon}_3) - \beta_1}{\varepsilon} \approx \frac{3.0492 - 3.0446}{0.01} = 0.4586 \tag{59}$$

By normalizing $(\alpha_{1,1}, \alpha_{1,2}, \alpha_{1,3})$, we obtain a unit vector of $(0.6538, 0.5718, 0.4955)$. Note that $\alpha_{1,2} = \alpha_{1,L_1}$ and $\alpha_{1,3} = \alpha_{1,L_2}$, the equivalent safety margin of component 1 is given by

$$G_1(\mathbf{U}) = \beta_1 + \alpha_{1,1}U_{1,1} + \alpha_{1,L_1}U_{L_1} + \alpha_{1,L_2}U_{L_2} = 3.0446 + 0.6538U_{1,1} + 0.5718U_{L_1} + 0.4955U_{L_2} \tag{60}$$

The partial safety factors $\gamma_{1,1}$ for load L_1 is then calculated by

$$\gamma_{1,1} = \frac{F_{L_1}^{-1}\left(\Phi(-\beta_1\,\alpha_{1,L_1})\right)}{\mu_{L_1}} = \frac{27.3884}{30} = 0.9129 \tag{61}$$

Similarly, the partial safety factors $\gamma_{1,2}$ for load L_2 is calculated by

$$\gamma_{1,2} = \frac{F_{L_2}^{-1}\left(\Phi(-\beta_1\,\alpha_{1,L_2})\right)}{\mu_{L_2}} = \frac{284.9148}{300} = 0.9497 \tag{62}$$

Then the supplier of component 1 provides $p_{f1} = 1.1650 \times 10^{-3}$, $\gamma_{1,1} = 0.9129$, and $\gamma_{1,2} = 0.9497$ to system designers.

Component 2 also has two limit-state functions given by

$$g_{2,1}(\mathbf{X}_2) = 2X_{2,2}^2 - 3X_{2,3} - 17X_{2,4} \tag{63}$$

$$g_{2,2}(\mathbf{X}_2) = X_{2,2}^2 - 2X_{2,3} - X_{2,4} \tag{64}$$

The independent basic random variables are $\mathbf{X}_2 = (X_{2,2}, X_{2,3}, X_{2,4}) = (L_1, L_2, X_{2,4})$, in which (L_1, L_2) are the same shared loads as those in FM1. Their distributions are given in Table 2.

Place Table 2 here

The supplier of component 2 uses FORM to conduct reliability analysis for both failure modes. They obtain the component probability of failure

$$p_{f2} = 1 - \Phi_2(-\boldsymbol{\mu}_2; \boldsymbol{\Sigma}_2) = 6.8864 \times 10^{-4} \tag{65}$$

in which $\boldsymbol{\mu}_2 = (-\beta'_{2,1}, -\beta'_{2,2}) = (-3.2559, -3.2848)$ and

$\boldsymbol{\Sigma}_2 = \begin{bmatrix} 1 & 0.9799 \\ 0.9799 & 1 \end{bmatrix}$.

The equivalent reliability index is given by

$$\beta_2 = -\Phi^{-1}(p_{f2}) = 3.1994 \tag{66}$$

The partial safety factor $\gamma_{2,1}$ for load L_1 is calculated by

$$\gamma_{2,1} = \frac{F_{L_1}^{-1}\left(\Phi(-\beta_2 \alpha_{2L_1})\right)}{\mu_{L_1}} = \frac{25.3615}{30} = 0.8454 \tag{67}$$

and $\gamma_{2,2}$ for load L_2 is given by

$$\gamma_{2,2} = \frac{F_{L_1}^{-1}\left(\Phi(-\beta_2 \alpha_{2L_2})\right)}{\mu_{L_2}} = \frac{307.0055}{300} = 1.0234 \tag{68}$$

Then $p_{f2} = 6.8864 \times 10^{-4}$, $\gamma_{2,1} = 0.8454$, and $\gamma_{2,2} = 1.0234$ are provided to system designers.

7.1.2 System-level analysis

To calculate the system reliability, system designers need to find the joint PDF of components 1 and 2. As discussed in Sec. 4, the joint PDF follows a multivariate normal distribution. The task of the system designers is therefore to find the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. With the given p_{f1} and p_{f2}, $\boldsymbol{\mu}$ is obtained by

$$\boldsymbol{\mu} = (-\Phi^{-1}(p_{f1}), -\Phi^{-1}(p_{f2})) = (-3.0446, -3.1994) \tag{69}$$

and $\boldsymbol{\Sigma}$ is determined by

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & (\alpha_{1,L_1}, \alpha_{1,L_2})(\alpha_{2,L_1}, \alpha_{2,L_2})^T \\ (\alpha_{1,L_1}, \alpha_{1,L_2})(\alpha_{2,L_1}, \alpha_{2,L_2})^T & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.4442 \\ 0.4442 & 1 \end{bmatrix} \tag{70}$$

where $\alpha_{i,L_j} (i, j = 1, 2)$ is calculated by

$$\alpha_{1,L_1} = -\beta_1^{-1}\Phi^{-1}\left(F_{L_1}(\gamma_{1,1}\mu_{L_1})\right) = 0.5718 \tag{71}$$

$$\alpha_{1,L_2} = -\beta_1^{-1}\Phi^{-1}\left(F_{L_2}(\gamma_{1,2}\mu_{L_2})\right) = 0.4955 \tag{72}$$

$$\alpha_{2,L_1} = -\beta_2^{-1}\Phi^{-1}\left(F_{L_1}(\gamma_{2,1}\mu_{L_1})\right) = 0.9665 \tag{73}$$

$$\alpha_{2,L_2} = -\beta_2^{-1}\Phi^{-1}\left(F_{L_2}(\gamma_{2,2}\mu_{L_2})\right) = -0.2190 \tag{74}$$

Thus the system probability of failure is given by

$$p_{fs} = 1 - \Phi_2(-\boldsymbol{\mu}; \boldsymbol{\Sigma}) = 1.8206 \times 10^{-3} \tag{75}$$

7.1.3 Result validation

To validate the result from the PSF method, we calculate the true p_{fs} using MCS method as if all the component design details, including all the component limit-state functions and the information in Tables 1 and 2, were available. For comparison, we also compute p_{fs} using the independence assumption method, which is given by

$$p_{fs,ind} = 1-(1-p_{f1,\mathrm{ind}})\times(1-p_{f2,\mathrm{ind}}) \quad (76)$$

in which

$$p_{f1,\mathrm{ind}} = 1-(1-p_{f11})\times(1-p_{f12}) = 1.1982\times10^{-3} \quad (77)$$

$$p_{f2,\mathrm{ind}} = 1-(1-p_{f21})\times(1-p_{f22}) = 1.0751\times10^{-3} \quad (78)$$

Plugging Eqs. (77) and (78) into Eq. (76), we have

$$p_{fs,ind} = 2.2721\times10^{-3} \quad (79)$$

The results from different methods are summarized in Table 3, which indicates that the PSF method produces much higher accuracy than the independence assumption method. The accuracy is measured by the relative error with respect to the MCS solution. The dependency between components is automatically accommodated in the proposed method. The large error from independence assumption method is mainly caused by the high correlation between component states.

Place Table 3 here

7.2 An engineering example

A hoisting device has two components as shown in Fig. 2. Component 1 consists of two ropes. Two loads L_1 and L_2 are applied to Component 1. L_1 and L_2 are independent, and the mean value of L_2 is much bigger than that of L_1. Component 2 is a truss structure and is composed of two rods. Components 1 and 2 are designed and manufactured by two independent outside suppliers, and no design details are available to the system-level analysis. System designers request the component suppliers to perform reliability analysis under the system loads L_1 and L_2 and to provide component reliabilities and PSFs for the loads.

Place Figure 2 here

7.2.1 Component-level analysis

Component 1 has two failure modes due to failures of ropes 1 and 2. The corresponding two limit-state functions are given by

$$g_{1,1}(\mathbf{X}_1) = S_1 - \frac{L_1 + L_2}{\pi d_1^2 / 4} \tag{80}$$

$$g_{1,2}(\mathbf{X}_1) = S_2 - \frac{L_2}{\pi d_2^2 / 4} \tag{81}$$

The distributions of random variables known by the suppliers of Component 1 are given in Table 4, in which *LogN* stands for a lognormal distribution.

Place Table 4 here

The supplier of Component 1 uses FORM for FM1 and then obtains the reliability index $\beta'_{1,1} = 3.8555$, and directional cosines $\alpha'_{1,1L_1} = -0.1604$ and $\alpha'_{1,1L_2} = -0.9451$ with respect to L_1 and L_2, respectively. The probability of failure is computed by $p_{f11} = \Phi(-\beta'_{1,1}) = 5.7744 \times 10^{-5}$.

The supplier then uses SORM for FM2 and obtains $\beta'_{1,2} = 3.2802$, $\alpha'_{1,2L_1} = 0$, and $\alpha'_{1,2L_2} = -0.6989$. The probability of failure is then given by $p_{f1,2} = \Phi(-\beta'_{1,2}) = 5.1873 \times 10^{-4}$.

The joint PDF of FM1 and FM2 is then determined by the mean

$$\boldsymbol{\mu}_1 = (-\beta'_{1,1}, -\beta'_{1,2}) = (-3.8555, -3.2802) \tag{82}$$

and the covariance matrix

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} 1 & 0.6604 \\ 0.6604 & 1 \end{bmatrix} \tag{83}$$

Thus, the probability of failure of Component 1 is calculated by

$$p_{f1} = 1 - \Phi_2(-\boldsymbol{\mu}_1; \boldsymbol{\Sigma}_1) = 5.6356 \times 10^{-4} \tag{84}$$

The corresponding reliability index is $\beta_1 = -\Phi^{-1}(p_{f1}) = 3.2567$.

Based on the equivalent limit-state function of Component 1, the PSFs $\gamma_{1,1}$ for L_1 and $\gamma_{1,2}$ for L_2 are calculated and are given by $\gamma_{1,1} = 1.0064$ and $\gamma_{1,2} = 1.4418$. The supplier then provides p_{f1} , $\gamma_{1,1}$ and $\gamma_{1,2}$ to system designers.

The two failure modes of Component 2 are caused by excessive axial stresses developed in Rods 1 and 2. The limit-state functions for the two failure modes are given by

$$g_{2,1}(\mathbf{X}_2) = S_3 - \frac{8(L_1 + L_2)a_2}{\sqrt{a_2{}^2 - a_1{}^2}\left(\pi d_3^2\right)} \tag{85}$$

$$g_{2,2}(\mathbf{X}_2) = S_4 - \frac{8(L_1 + L_2)a_1}{\sqrt{a_2{}^2 - a_1{}^2}\left(\pi d_4^2\right)} \tag{86}$$

The distributions of random variables are given in Table 5.

Place Table 5 here

The supplier of Component 2 applies FORM to both failure modes and obtains the reliability index and directional cosines for the loads. For FM1, the supplier obtains $\beta'_{2,1} = 2.7199$, $\alpha'_{2,1L_1} = -0.1907$, and $\alpha'_{2,1L_2} = -0.9299$. For FM2, the results are $\beta'_{2,2} = 2.8845$, $\alpha'_{2,2L_1} = -0.1789$, and $\alpha'_{2,2L_2} = -0.8697$. The probability of failure is then calculated by $p_{f2} = 1 - \Phi_2(-\boldsymbol{\mu}_2; \boldsymbol{\Sigma}_2) = 4.3144 \times 10^{-3}$, and the reliability index is $\beta_2 = -\Phi^{-1}(p_{f2}) = 2.6264$. The PSFs $\gamma_{2,1} = 1.0610$ for load L_1 and $\gamma_{2,2} = 1.4506$ for load L_2 are also obtained. The supplier then provides p_{f2}, $\gamma_{2,1}$, and $\gamma_{2,2}$ to system designers.

7.2.2 System-level analysis

With the provided component probabilities of failure p_{f1}, and p_{f2}; PSFs $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{2,1}$ and $\gamma_{2,2}$ for the system loads, system designers build the joint CDF of Components 1 and 2, which follows a multivariate normal distribution with the mean

$$\boldsymbol{\mu} = (-\Phi^{-1}(p_{f1}), -\Phi^{-1}(p_{f2})) = (-3.2567, -2.6264) \tag{87}$$

and the covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & (\alpha_{1,L_1}, \alpha_{1,L_2})(\alpha_{2,L_1}, \alpha_{2,L_2})^T \\ (\alpha_{1,L_1}, \alpha_{1,L_2})(\alpha_{2,L_1}, \alpha_{2,L_2})^T & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.7072 \\ 0.7072 & 1 \end{bmatrix} \tag{88}$$

where $\alpha_{i,L_j}\ (i, j = 1, 2)$ is given by

$$\alpha_{1,L_1} = -\beta_1^{-1}\Phi^{-1}\left(F_{L_1}(\gamma_{1,1}\mu_{L_1})\right) = -0.0163 \tag{89}$$

$$\alpha_{1,L_2} = -\beta_1^{-1}\Phi^{-1}\left(F_{L_2}(\gamma_{1,2}\mu_{L_2})\right) = -0.7462 \tag{90}$$

$$\alpha_{2,L_1} = -\beta_2^{-1}\Phi^{-1}\left(F_{L_1}(\gamma_{2,1}\mu_{L_1})\right) = -0.1937 \tag{91}$$

$$\alpha_{2,L_2} = -\beta_2^{-1}\Phi^{-1}\left(F_{L_2}(\gamma_{2,2}\mu_{L_2})\right) = -0.9435 \tag{92}$$

Thus the system probability of failure is given by

$$p_{fs} = 1 - \Phi_2(-\boldsymbol{\mu}; \boldsymbol{\Sigma}) = 4.6386 \times 10^{-3} \tag{93}$$

7.2.3 Result validation

We also calculate the true system probability of failure using MCS method and the independence assumption method, and the results are shown in Table 6.

The results show that the proposed method outperforms the independence assumption method with a relatively higher accuracy even with limited information available for system reliability analysis.

Place Table 6 here

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8. Conclusions

This works develops a new system reliability method to accurately estimate product reliability with only component reliability and partial safety factors for shared system loads. The new method provides a solution to the challenge for accurate system reliability prediction when component design details are not available to system designers due to outsourcing. Two case studies demonstrate that the proposed method is more accurate than the traditional independence assumption method that neglects the dependence between component states. The new strategy is for system designers to construct equivalent component limit-state functions using the partial safety factors for shared system loads provided by component suppliers. Then the joint probability density function of all the component states is obtained at the system level, thereby leading to an accurate system reliability prediction without revealing proprietary details of outsourced components.

The proposed method is applicable to series systems whose failures are caused by excessive stresses due to random loads. The major assumption of the proposed method is that the shared loads are only common random variables between the components of the system. If there are other common variables, the proposed method can still work as long as the corresponding partial safety factors are produced and provided by component suppliers.

The proposed method can be extended to other system configurations, including parallel systems, mixed systems, and linked networks. Equivalent component limit-state functions can be generated the same way, but the final calculation of the system reliability will be different. The joint probability of component failures for the system reliability estimation will depends on the specific system configuration or the logic relationships between the system and component states.

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Table 1 Distribution of basic random variables

Variable	Distribution
$X_{1,1}$	$N\left(10, 0.8^2\right)$
$X_{1,2}(L_1)$	$N\left(30, 1.5^2\right)$
$X_{1,3}(L_2)$	$N\left(300, 10^2\right)$

Table 2 Distribution of basic random variables

Variable	Distribution
$X_{2,2}(L_1)$	$N\left(30, 1.5^2\right)$
$X_{2,3}(L_2)$	$N\left(300, 10^2\right)$
$X_{2,4}$	$N\left(20, 2^2\right)$

Table 3 Results from different methods

	PSF	Independence Assumption	True Value
p_{fs}	1.8206×10^{-3}	2.2721×10^{-3}	1.7689×10^{-3}
Error (%)	2.92	28.44	N/A

Table 4 Distribution of basic random variables

Random variables	Distribution
$X_{1,1}$ (d_1) : diameter of rope 1	$N\left(5\times10^{-3},(1\times10^{-4})^2\right)$ m
$X_{1,2}$ (d_2) : diameter of rope 2	$N\left(4\times10^{-3},(1\times10^{-4})^2\right)$ m
$X_{1,3}$ (S_1) : resistance of rope 1	$N\left(70,1^2\right)$ MPa
$X_{1,4}$ (S_2) : resistance of rope 2	$N\left(95,12^2\right)$ MPa
$X_{1,5}$ (L_1) : load 1	$logN\left(250,30^2\right)$ N
$X_{1,6}$ (L_2) : load 2	$logN\left(550,100^2\right)$ N

Table 5 Distribution of basic random variables

Random variables	Distribution
$X_{2,1}$ (a_1) : length of Rod 1	$N\left(0.9,(1\times10^{-4})^2\right)$ m
$X_{2,2}$ (a_2) : length of Rod 2	$N\left(1.8,(1\times10^{-4})^2\right)$ m
$X_{2,3}$ (d_3) : diameter of Rod 1	$N\left(6\times10^{-3},(1\times10^{-4})^2\right)$ m
$X_{2,4}$ (d_4) : diameter of Rod 2	$N\left(6\times10^{-3},(1\times10^{-4})^2\right)$ m
$X_{2,5}$ (S_3) : resistance of Rod 1	$N\left(95,3^2\right)$ MPa
$X_{2,6}$ (S_4) : resistance of Rod 2	$N\left(50,3^2\right)$ MPa
$X_{2,7}$ (L_1) : load 1	$logN\left(250,30^2\right)$ N
$X_{2,8}$ (L_2) : load 2	$logN\left(550,100^2\right)$ N

Table 6 Results from different methods

	PSF	Independence Assumption	True Value
p_{fs}	4.6386×10^{-3}	5.7917×10^{-3}	4.8540×10^{-3}
Error (%)	4.43	19.32	N/A

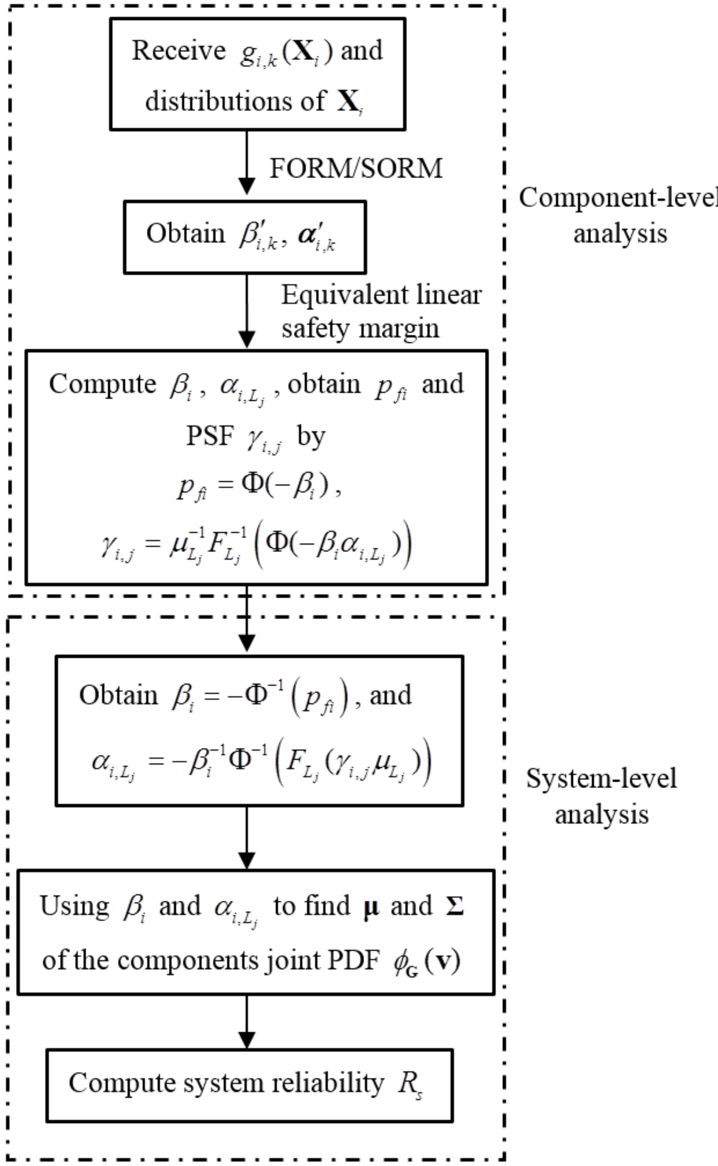

Fig. 1 Flowchart of the proposed method

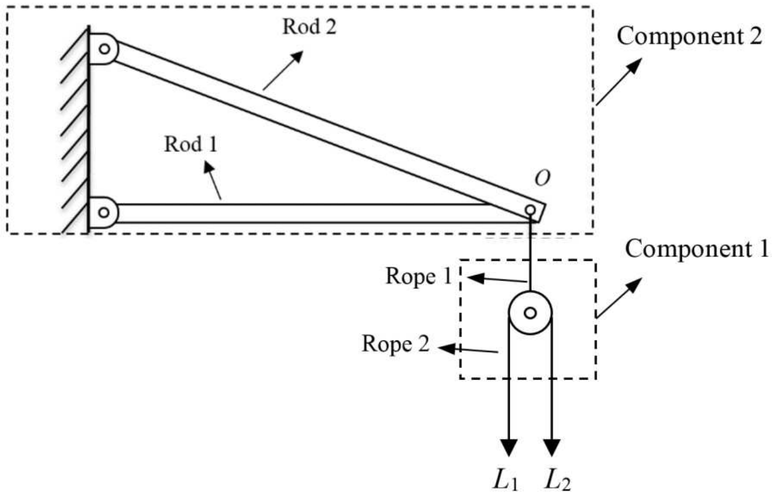

Fig. 2 A hoisting device